GIVING PROGNOSIS FOR THE CHOSEN FEATURES OF THE OPERATION AND MAINTENANCE SYSTEM BASED ON MODEL INVESTIGATIONS

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Abstract

The paper deals with the chosen issues related to modelling, giving prognosis and controlling maintenance process of a certain class of the technical objects being realized in a complex maintenance system. Supporting a decision maker in the decision making process related to the analysed maintenance system is to forecast behaviour of the maintenance system and evaluate influence of the chosen decision making variants on the maintenance process course. The object of the investigation, being the basis to illustrate the considerations presented herein, is an urban bus maintenance system in a selected urban agglomeration. The purpose of the paper is to present a possibility to use the Markov's model of the technical object maintenance process to preliminarily forecast the maintenance system state after changing the model input parameter values. The change of the model input parameter values may simulate influence of the internal and external factors on the system behaviour. The presented method to model and forecast a maintenance system due to the assumed generalization degree of the description and the system approach to the problem may be used to forecast and analyse a maintenance process being carried out in other maintenance systems than the one in an urban bus transport system.

Keywords: *Markov's model, maintenance process, forecasting, urban public transport*

1. Introduction

The system under analysis is an urban bus transport maintenance system. The urban bus transport maintenance process is characterised by cyclic changes of the maintenance phases. For the decision makers of the system in which such a process is realized it is of utmost importance to have the tools (with various complexity degree) enabling to analyse and forecast the system state and facilitating to make the decisions concerning the control of the process being realized.

The elements supporting a decision maker in the process of making decisions as to the control of the maintenance process and the system in which they are realized may be the investigation results of maintenance process models and computer programs to simulate this process course. Of course it is also needed to take into account the economic criterion which was not an aim of the considerations when performing the analyses.

A homogenous Markov's process was applied as a mathematical model of changes of the maintenance states of the technical objects.

The paper presents assumptions to build a model of the process realized in the investigation object and the method to analyse the model and forecast the system state. All the considerations are illustrated on the basis of a simplified (for the paper purposes) four-state model of the maintenance process.

Supporting a decision maker in the process of forecasting the state of the maintenance system under analysis and making decisions concerning the control of the system is to forecast influence of the chosen decision making variants on the course and effectiveness of the maintenance process. It may be achieved by analysing the results of the maintenance process model investigations to assess the model parameter values (corresponding to the decision making variants under analysis) and to determine, in each case, selected measures of technical effectiveness of the realized process.

2. Investigation object

The investigation object is an urban bus transport maintenance system in a selected urban agglomeration, while the investigation subject is a selected set of maintenance states of the means of transport and their maintenance process, including the interrelations occurring between the above elements and between them and the maintenance process effectiveness.

When operating and maintaining busses various events occur, the effects of which affect the bus use and service processes, as well as their technical states and economic result of the system operation in which they are used. The urban transport buses may be in various maintenance states forming the state space S in the maintenance process.

A bus transport system performs tasks in sequential maintenance phases creating a working cycle (Fig. 1). The following maintenance phases creating a working cycle have been determined: f_1 – element activation phase, f_2 – task realization phase, f_3 – servicing phase (after completing the daily transport tasks), f_4 – waiting phase to include a bus in the use process (so called organizational stop).

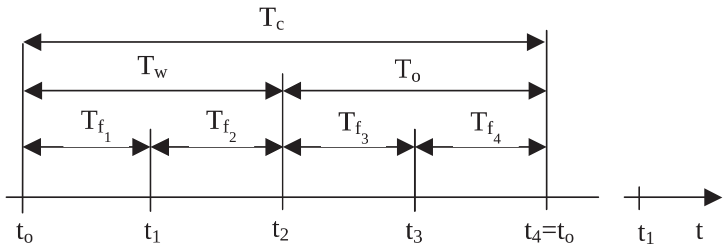

Fig. 1. Working cycle: T_w – time for which a technical object stays in an executive subsystem (process one), T_o – time for which a technical object stays in a servicing subsystem (a service station or a bus depot), T_c – duration of one cycle

The individual elementary subsystems of <H-TO> type (driver-bus) realize the transport tasks assigned to them in the task realization phase f_1. Due to a possibility of occurrence of a damage to a bus or a driver's inability state, the transport task realization time for the individual elementary subsystems is of a random nature.

A damaged bus in the investigated maintenance system is directed to the serviceability assurance subsystem, where it is a subject to servicing process. Once the servicing processes are finished a task-usable vehicle is directed to realize the tasks or to so called bus depot (if the vehicle cannot undertake the task due to the transport task schedule applied in the system). The vehicle servicing duration is of a random nature.

In order to restore, as quickly as possible, serviceability of the vehicles which got damaged when realizing their tasks, so called technical emergency service units are used. The scope of the services performed by the operators of the technical emergency service units is limited by the technical equipment available for those units and by the necessity to perform the services outside the service station facilities. The time of restoring the serviceability is of a random nature. A vehicle whose serviceability has been restored as a result of actions of the operators of the technical emergency service units undertakes the discontinued realization of its task.

It is important, in the aspect of assuring continuous realization of the transport tasks and the technical and economic effectiveness achieved by the system, so that as many as possible vehicles are in the task-ability state and realize the transport tasks in the time of the working cycle phase f_1. It is influenced by such factors as: features of the vehicles in use, type of the equipment and number of the service stands in a service station and the number of the technical emergency service units, diagnostic tools used by the technical emergency service units, nature of the transport tasks, method and scope of performing the service processes and others.

So, there is a possibility to have influence on the system ability to realize the undertaken transport tasks. Therefore, it is needed to evaluate (forecast) influence of the actions undertaken in the system on the maintenance process course and the system behaviour.

3. Assumptions for the maintenance process model

The assumptions for the maintenance process model are determined on the basis of identification of the investigation object and the maintenance investigations performed as well as on the basis of the analysed space of the maintenance states and the events concerning the buses used in the investigation object. The model has been described in the category of the maintenance states and probability of the state changes.

Due to identifying the object of the investigation it is possible to determine *n* of the bus maintenance states being significant for the paper purpose. The maintenance states are characterised by their duration time distributions. The stochastic process $\{X(t), t \geq 0\}$ is a mathematical model of the bus maintenance process. The analysed stochastic process $\{X(t), t \geq 0\}$ has a finite phase space S, $S=\{S_1, S_2, ..., S_n\}$. It was assumed that the work of the model is described by the homogenous Markov's process $\{X(t) : t \in R_+\}$ with a finite set of the states S. The states of the analysed stochastic process correspond to the determined bus maintenance states.

Each of the technical objects used may, at a given moment of time t, be only in one of the determined states. Staying in the individual states generates incomes or/and expenses related to the system operation.

A realization of the process is a sequence of the determined states. The sequence of the states, duration of the individual states and their occurrence frequency depend mostly on the specific features of the technical objects, the features of the processes affecting them and on the features and structure of the subsystems working together in the maintenance process realization.

The following bus maintenance states being significant for the analysis of the operation effectiveness of the investigated system and for forecasting the system state after changing the controlling interactions were determined for the purposes of this paper:

S_1 - state of using – a state in which a bus including its operator realizes the assigned transport tasks,

S_2 - state of servicing performed in the maintenance system environment – a state in which a bus is a subject to corrective service processes performed by the units of so called technical emergency service due to occurrence of a damage to a vehicle when realizing the transport tasks,

S_3 - state of servicing performed in the serviceability assurance subsystem which occurs, for instance, when there was a damage that could not be removed outside the service station by the technical emergency service units,

S_4 - state of waiting for realization of the transport tasks.

On the basis of identification of the real urban bus transport maintenance system the possible transitions between the determined bus maintenance states were specified, as illustrated in the Fig. 2.

For the determined maintenance states a matrix of the process state intensity change $\{X_t, t \in T\}$ was assessed on the basis of the investigations performed in the real maintenance system.

$P_i(t) = P\{X(t)=S_i\}$ was used to identify probability that the process X(t) is in the state $S_i \in S$ at the moment of time t. It was assumed that the initial state of the process X(t) is the state S_1, it means that the initial distribution of the analysed process takes the form:

$$P\{X(0)=S_1\} = 1, \tag{1}$$

$$P\{X(0)=S_i\} = 0 \qquad \text{for } i \neq 1, S_i \in S. \tag{2}$$

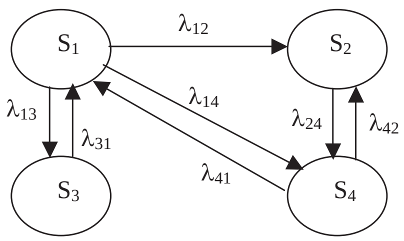

Fig. 2. Directed graph of the maintenance states

The intensity $\lambda_{i,j}$, i, j = 1, 2, 3, 4 of changes of the process state $\{X_t, t \in T\}$ from the state $S_i \in S$ to the state $S_j \in S$ was included in so called transition intensity matrix Λ:

$$\Lambda = \begin{vmatrix} -\lambda_{11} & \lambda_{12} & \lambda_{13} & \lambda_{14} \\ 0 & -\lambda_{22} & 0 & \lambda_{24} \\ \lambda_{31} & 0 & -\lambda_{33} & 0 \\ \lambda_{41} & \lambda_{42} & 0 & -\lambda_{44} \end{vmatrix}. \tag{3}$$

The following designations were introduces in order to simplify the formula:

$$\lambda_{11} = \lambda_{12} + \lambda_{13} + \lambda_{14}, \tag{4}$$

$$\lambda_{22} = \lambda_{24}, \tag{5}$$

$$\lambda_{33} = \lambda_{31}, \tag{6}$$

$$\lambda_{44} = \lambda_{41} + \lambda_{42}. \tag{7}$$

By using the system of differential equations by A. N. Kolmogorov [2, 3, 8] it is possible to determine the probabilities $P_i(t)$ that the Markov's process $\{X(t), t \geq 0\}$ stays in the analysed states S_i at the moment of time t, it means $P_i(t) = P\{X(t) = S_i\}$.

The transition intensity matrix Λ allows to create the following system of differential equations:

$$\begin{aligned} P_1^{'}(t) &= -\lambda_{11}P_1(t) + \lambda_{31}P_3(t) + \lambda_{41}P_4(t), \\ P_2^{'}(t) &= \lambda_{12}P_1(t) - \lambda_{22}P_2(t) + \lambda_{42}P_4(t), \\ P_3^{'}(t) &= \lambda_{13}P_1(t) - \lambda_{33}P_3(t), \\ P_4^{'}(t) &= \lambda_{14}P_1(t) + \lambda_{24}P_2(t) - \lambda_{44}P_4(t). \end{aligned} \tag{8}$$

In order to achieve an unambiguous solution of the system of equations (8), it is needed to adopt the initial conditions determined with the relations (1) and (2).

For the analysed process $\{X(t), t \geq 0\}$ there is a stationary distribution of the process which does not depend on the process initial distribution:

$$\lim_{t\to\infty} P_i(t) = p_i^*. \tag{9}$$

The stationary probabilities p_i^* meet the system of equations [8]:

$$\begin{cases} \sum_{i=1}^{n} p_i^* \lambda_{ij} = 0, \quad \text{dla} \quad j = 1,2,3,\ldots,n, \\ \sum_{i=1}^{n} p_i^* = 1. \end{cases} \tag{10}$$

On the basis of the determined matrix Λ and the normalizing condition, the system of equations (10) may be formulated as:

$$\begin{cases} -\lambda_{11}p_1^* + \lambda_{31}p_3^* + \lambda_{41}p_4^* = 0, \\ \lambda_{12}p_1^* - \lambda_{22}p_2^* + \lambda_{42}p_4^* = 0, \\ \lambda_{13}p_1^* - \lambda_{33}p_3^* = 0, \\ p_1^* + p_2^* + p_3^* + p_4^* = 1. \end{cases} \tag{11}$$

The solution of the system of equations (11) will take the following form:

$$
\begin{aligned}
p_1^* &= \frac{1}{b}, \\
p_2^* &= \frac{a}{b}, \\
p_3^* &= \frac{\lambda_{13}}{\lambda_{31} b}, \\
p_4^* &= \frac{\lambda_{12} + \lambda_{14}}{\lambda_{411} b},
\end{aligned}
\tag{12}
$$

where:

$$
a = \frac{\lambda_{12} + \lambda_{42}\left(\dfrac{\lambda_{12} + \lambda_{14}}{\lambda_{41}}\right)}{\lambda_{24}}, \tag{13}
$$

$$
b = 1 + a = \frac{\lambda_{13}}{\lambda_{31}} + \frac{\lambda_{12} + \lambda_{14}}{\lambda_{41}}. \tag{14}
$$

3.1. Extending the state space

In case, when the distributions of the duration times S_i, $i = 1, 2, ..., n$, $S_i \in S$ of the process $\{X(t), t \geq 0\}$ are not exponential distributions. it is possible to transform this process into the process $\{Y(t), t \geq 0\}$ with a finite state space $U = \{U_1, U_2, ..., U_k\}$, $k > n$ for which the distributions of the state duration times will be exponential distributions. The further part of the paper presents the way to transform the process, when the distribution of a random variable is a subject to the Erlang distribution. Then the random variables T_i, $i = 1, 2, ..., n$ may be presented as [15]:

$$
T_i = T_{i1} + T_{i2} + ... + T_{is}, \tag{15}
$$

where T_{ij} - independent random variables of an exponential distribution with the parameter λ, $i = 1, 2, ..., n$, $j = 1, 2, ..., s$.

To illustrate the investigations it was assumed that the random variable T_1 has Erlang distribution. The state S_1, whose duration time is a random variable with Erlang distribution was replaced by the states $U_1, U_2, ..., U_s$. This way the process X(t) with the space of states S and with non-exponential distribution of the state duration S_1 was transformed into the process Y(t) with the state space U characterised by exponential times of the state duration.

It is only possible to go from the state U_{1j} $(1 \leq j \leq s-1)$ to the state $U_{1\ j+1}$. The graphic interpretation of the transformation (decomposition) of the state S_1 is shown in the Fig. 3.

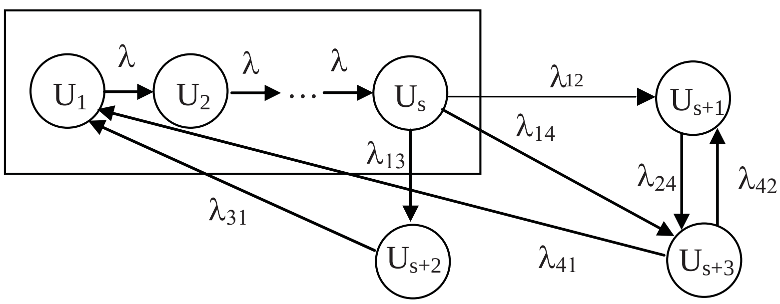

Fig. 3. Directed graph representing transformation of the state U_1

The Erlang distribution is characterised by the following cumulative distribution function:

$$F(x) = 1 - e^{-\lambda x}\left\{1 + \frac{\lambda x}{1!} + \frac{(\lambda x)^2}{2!} + \ldots + \frac{(\lambda x)^{s-1}}{(s-1)!}\right\}, \tag{16}$$

$$x \geq 0,\ \lambda > 0,\ s \in N_+ . \tag{17}$$

As a criterion of the optimal selection of the parameter λ and k it is possible to adopt the minimum sum of squares of the deviation of the cumulative distribution function value of the Erlang distribution F(x) from the empiric cumulative distribution function $F_e(x)$:

$$S(\lambda, s) = \sum_{l=1}^{m}\left[F(x_l) - F_e(x_l)\right]^2 . \tag{18}$$

This is the way to determine the number s of the states U_i, i = 1, 2, ..., s.

4. Giving prognosis for the selected system features

Supporting a decision maker in the decision making process may be achieved by analysing the results of the maintenance process model investigations to assess the values of the model parameters (corresponding to the analysed decision making variants). A change of the value of the model initial parameters may simulate influence of the internal and external factors on the system behaviour.

It is possible, for instance, to forecast influence of a change of the applied technical emergency service unit type (the technical emergency service units different by their type and by the technical equipment being crucial for the scope of the repairs performed and the servicing duration are applied in the analysed maintenance system) on the process course and the system behaviour. Another example may be an attempt to assess influence of the transport task type (in the urban transport systems described by the type of so called transport route) on the course of the analysed process. The transport task realization conditions have influence, among other things, on: type and frequency of the damages, costs related to the task realization, fuel consumption, etc. (resulting from the road condition, number of passengers, route length, number of bus stops, etc.).

4.1. Computational example

A hypothetical, simplified computational example was prepared to illustrate the investigations. Effects of replacing the technical equipment (with more modern one) of the service stands and of the technical emergency service units were simulated. This change resulted in shortened duration time of the processes to bring back the vehicle serviceability. The change of the value of the expected random variable expressing the servicing duration time (both at the service station stands and of the technical emergency service units) may also be a result of interaction of other factors such as changing the organization and conditions to realize the servicing processes, increasing the number of operators of the technical emergency service units, etc.

Two calculation variants were prepared. The calculations prepared for the values of the model parameters corresponding to the state preceding the effects of the simulated event were denoted with „sym A”, while the variant of the calculations simulating the aforementioned event was denoted as „sym B”.

Tab. 1. Essential input data used to make the calculations

	λ_{12}	λ_{13}	λ_{14}	λ_{24}	λ_{31}	λ_{41}	λ_{42}
Sym A	0.0586	0.0521	0.0509	2.6324	0.8125	4.6345	0.4087
Sym B	0.0586	0.0521	0.0509	3.5325	2.1365	6.3814	0.3126

The essential data used for calculations are presented in the Table 1. The Table 2 shows the chosen results of calculating the limiting probabilities for the individual calculation variants (determined on the basis of the relation (12)). The calculations were made by means of the developed computer computational programs.

The analysis of the changes of the probability values $P_i(t)$, for the both calculation variants, proves that these probabilities are established after some time and reach the value corresponding to the value of the limiting probabilities p_i^*.

Tab 2. Calculation results of the stationary probability values

	p_1^*	p_2^*	p_3^*	p_4^*
Sym A	0.8977	0.0236	0.0578	0.0209
Sym B	0.9430	0.0166	0.0237	0.0167

The results of the performed model investigations confirm the expected reactions of the model to the changes of their parameters. It proves that the calculations are correct and that the model is useful to prepare preliminary prognosis for the state of the analysed system after changing the levels of influence of the investigated factors on the system.

5. Summary

The purpose of the paper was to present a possibility to use the Markov's model of the technical object maintenance process for giving preliminary prognosis for the maintenance system state after changing the controlling forcing factors (model input parameter values). A change of the model input parameter value simulates influence of the internal and external factors of the system behaviour.

The considered model of an urban bus transport maintenance process is significantly simplified. However, the presented method of building models of that type and their analyses prove that it is possible to use them to preliminarily forecast the system state.

The analysis of the maintenance process model investigation results, performed for the model input parameters assessed on the basis of the real maintenance data, makes it possible to evaluate the effects of mutual work of the essential processes (use) and auxiliary (service), moreover it supports a decision maker in the decision making process concerning the control of the maintenance process and system in which it is realized.

It seems that the analysis of the model investigation results for various model parameter values (decision making variants) assessed on the basis of the maintenance investigation results may be the basis to have influence on a real maintenance system in order to make the maintenance process realized in it more reasonable (after expanding the model in such a way to increase its adequacy to the real system, for instance by increasing the number of the analysed vehicle maintenance states).

The analysis of the results of the performed model investigations shows that the model is susceptible to the change of its input parameter values.

The assumed parametric nature of the model allows to adopt it easily to any possible changes occurring in a real maintenance system.

In case when on the basis of identification of a real maintenance process it is possible adopt the assumptions resulting from the use of the theory of homogenous Markov's processes (especially a lack of so called secondary action (process "memory") and constant in time matrix of intensity of the state changes) to model the analysed process, and the distributions of the durations time S_i, $i = 1, 2, ..., n$ of the process $(\{X(t), t \geq 0\})$ are not exponential distributions it is possible to transform this process to the process $\{Y(t), t \geq 0\}$ with finite state space $U = \{U_1, U_2, ..., U_k\}$, $k > n$ for which the distributions of the state duration times will be exponential distributions.

Due to the assumed degree of generality of description and system approach to the problem, the method to model and forecast a maintenance process presented herein may be used to forecast and analyse a maintenance process realized in other maintenance systems than the one in an urban bus transport system.

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